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A new approach to sparse decomposition of nonstationary signals with multiple scale structures using self-consistent nonlinear waves

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HIGHLIGHTS

- A discussion on the common assumptions in sparse decomposition methods.
- Analytical derivation and framework construction of a new sparse decomposition (SD) method.
- Establishment of the core concepts of the new SD approach and possible extensions.
- Performance comparison between the new SD and the empirical mode decomposition.

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ABSTRACT

A new method for signal decomposition is proposed and tested. Based on self-consistent nonlinear wave equations with self-sustaining physical mechanisms in mind, the new method is adaptive and particularly effective for dealing with synthetic signals consisting of components of multiple time scales. By formulating the method into an optimization problem and developing the corresponding algorithm and tool, we have proved its usefulness not only for analyzing simulated signals, but, more importantly, also for real clinical data.

1. Introduction

In scientific and engineering studies, we often attempt to extract information in certain oscillatory components in a given signal that are masked by other complex fluctuations. There are many situations in which the observed signal carries several components [1–3], each generated by a different physical source or mechanism. Therefore, it is very useful and often essential to decompose the data into components at different time scales. In light of this, the importance of sparse representations has recently been recognized. Sparse representations [4], in which a signal is casted into the sum of a few

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appropriate simpler functions, not only can make the computational work more efficient, but also can manifest the physical structures and substantial, if sometimes subtle, mechanisms. The Empirical Mode Decomposition (EMD) [5], for example, is a particularly well-known method of obtaining sparse representations for signals with oscillatory components at multiple time scales. In addition to EMD, there are many other approaches to decomposing signals into components at different time scales [6–9]. While each of those methods has its own merits, they all transform the problem into an optimal fitting problem with somewhat restrictive assumptions such as a presupposed number of components that are often introduced purely by mathematical considerations without clear physical meaning. Even for EMD in which there is no assumption about the spectral properties of a signal's components beforehand [10], the lacking of physical assumptions and guiding principles sometimes renders EMD not as effective as expected. In fact, many complex signals arising in natural sciences, in particular physiological signals, are composed of many ingredients from various sources or physical mechanisms, and those components would occasionally exhibit similar oscillatory behaviors at close frequencies over different time periods, while each having its own underlying mechanism and characteristics. In many cases what is really needed is the information contained in a single component. However, it is rather difficult to separate it from the rest using a purely time domain algorithm such as EMD [11]. This occurs because the other components cannot be simply viewed as disturbances or noises, and a direct 'attack' on this intricacy without physical guiding principles often leads to distortions of the wanted information (see the section on examples [11]). In this article, we present a new method of sparse decompositions of signals that bear multiple-scale oscillations. Our goal is to better analyze the results of a few simple and commonly adopted assumptions on the phases and the amplitudes (mainly Eq. (2) below) of the signals, and to provide a new theoretical framework that might be able (in the future) to analyze and characterize more precisely the features and the limitations of EMD and other sparse decomposition methods. The proposed method has a similar performance comparable to that of EMD in many situations, and exhibit valuable advantages in certain cases. The most essential difference between the new method and the aforementioned ones (except EMD) is that the assumptions made in the new method are describing regular physical conditions of self-sustaining mechanisms that underlie many observed signals (more details in Section 2). Therefore, this new approach will prove more natural physically, and, consequently, its application can most likely be extended well beyond the range of the initial assumptions.

2. Basic assumptions and signal model

Suppose the observed signal consists of several components with well-separated frequency ranges and each of them is a product of a positive slow varying envelope function and a oscillating function of unit magnitude. The oscillating function can be represented as a cosine function with smooth phase such that the derivative of the phase function is always assumed to be positive and has much greater magnitude than its second derivative and the derivative of the envelope (note that this assumption is deduced from commonly observed and agreed features of the intrinsic mode functions of EMD, see [12], so here we are taking it as our logical starting point). This assumption is generally true for signals generated by many physical "self-sustaining" systems [13], where it is often observed that the cycle length of a quasi-periodic oscillation is relatively slowly (compared with the phase varying of the oscillation) tuning, in a way that is characteristic of the system generating the oscillation but relatively robust and insensitive to external disturbances. A mathematically simplification of this assumption though being not entirely impractical is that, within a very short time interval, a signal, $S(t)$, can be viewed as the combination of a high frequency component and another function which is the sum of all other slow varying components, namely:

$$S(t) = a(t)\cos[\omega(t)\,t + \phi] + b(t), \quad t \in [t_0 - \Delta t, t_0 + \Delta t] \tag{1}$$

where

$$\omega(t) > 0, \qquad a(t) > 0, \qquad \omega(t) \gg a'(t), b'(t), \omega'(t). \tag{2}$$

Notice that the representation of the signal in Eq. (1) is not global, and it can only be a good approximation locally. Nonetheless, the point here is not to provide an exact and precise global formulation, but rather to derive an equation that characterizes approximately the relationship between the high frequency component and the original signal. The equation will in turn serve as the directing principle for an optimization problem that will model realistic situations accurately. As in the lowest order perturbation theory, in the small interval we are looking at, the signal is approximately:

$$S(t) = a(t_0)\cos[\omega(t_0)\,t] + b(t_0) \tag{3}$$

where the slow varying functions are replaced by constants. Next, we obtain the envelope equation at t_0 by differentiation and using the approximations we made (see, for example, [14]):

$$\left.\frac{dS(t, t_0)}{dt_0}\right|_{t_0=t} = 0$$

$$S'(t) + a(t)\sin[\omega(t)\,t]\cdot\omega(t) = 0. \tag{4}$$

Note that we are actually granting the approximation that

$$F'(t) \cong S'(t). \tag{5}$$

Remember, the wanted fast component is:

$$F(t) = a(t)\cos[\omega(t)\,t]. \tag{6}$$

Now, the equation can be put into:

$$[a(t)\sin\omega(t)\,t]^2 \cdot F''(t) + \left[S'(t)\right]^2 \cdot F(t) = 0. \tag{7}$$

Recall that, in general, for the Hilbert transform:

$$\begin{aligned} &\mathcal{H}\{x(t)\,y(t)\} \neq \mathcal{H}\{x(t)\}\,\mathcal{H}\{y(t)\} \\ &\mathcal{H}\{\cos\theta(t)\} \neq \sin\theta(t). \end{aligned} \tag{8}$$

The ideal and convenient case where

$$\mathcal{H}\{a(t)\cos\theta(t)\} = a(t)\sin\theta(t) \tag{9}$$

has been called the "mono-component" functions by some mathematicians. But this does not appear to be the standard terminology, and there are other definitions for being "mono-component". Nevertheless, we would like this property to hold at least approximately, that is, we assume:

$$\mathcal{H}\{a(t)\cos\theta(t)\} \cong a(t)\sin\theta(t). \tag{10}$$

This assumption will lead to the aimed equation later, and that equation will in turn render our dictionary for the optimal fitting problem consisting of those functions for which Eq. (10) approximately holds (note that this is not circular reasoning because we are translating a physical and mathematical property into a part of the objective of an optimization problem, so that it will survive to a large extent in our dictionary for decomposition). Of course, given a function of time, such a representation may not be readily available, and almost certainly not unique even if exists. The point here is that we want the members of our dictionary and, thus, the obtained components, to behave as described by Eq. (10) as much as possible. By imposing this property implicitly in our optimization constraints, such components are obtained. With all the approximations at hand (Eqs. (5) and (10)), we now have:

$$\{F(t)\}^2 \cdot F''(t) + \left[\mathcal{H}\left[S'(t)\right]\right]^2 \cdot F(t) = 0. \tag{11}$$

Mathematically, the use of the envelope equation is to "patch up" the approximations made around each instant locally in order to give a global differential equation. What we have done is quite similar to the resummation method used in the perturbation theory to improve convergence and range of applicability [15]. It would also be desired for the solved component to be orthogonal to the residual, namely, $\langle S(t) - F(t), F(t)\rangle \approx 0$. However, this condition might be relaxed to some extent in practice. Note that the equation is actually similar to the common wave equation, as we can put it as follows:

$$F''(t) + \left[\mathcal{H}\left[S'(t)\right]\right]^2 / \{F(t)\}^2 \cdot F(t) = 0. \tag{12}$$

Theoretically, the zeros of $F(t)$ would not cause any problem because it can be proved that under our assumptions the coefficient always has a finite well-defined limit when $F(t)$ is zero. However, of course, this will not always hold in real applications, and might cause numerical errors. Fortunately, there is a more stable equivalent quantity that can characterize our "instantaneous frequency":

$$\omega^2(t) = \frac{\left[S'(t)\right]^2 + \left[\mathcal{H}\left[S'(t)\right]\right]^2}{\{F(t)\}^2 + [\mathcal{H}[F(t)]]^2} = \frac{\left[F'(t)\right]^2 + \left[\mathcal{H}\left[F'(t)\right]\right]^2}{\{F(t)\}^2 + [\mathcal{H}[F(t)]]^2}, \tag{13}$$

and therefore, the equation is turned into:

$$F''(t) + \omega^2(t) \cdot F(t) = 0. \tag{14}$$

In fact, we will use the rightmost term in Eq. (13) as our working definition of the instantaneous frequency, which will also be compared with the traditional definition (defined using Hilbert transform and inverse trigonometric functions) later and will prove to be often computationally more stable and realistic. In the derivation of the above equation, it has been assumed that the magnitude of $\omega'(t)$ is so small that it is negligible compared with $\omega(t)$. If, however, this assumption does not hold well, then Eq. (14) should be replaced by the following equation:

$$F''(t) - \frac{\omega'(t)}{\omega(t)}F'(t) + \omega^2(t) \cdot F(t) = 0. \tag{15}$$

However, Eq. (14) suffices for the applications in this paper, performing well on both simulated and actual clinical data, and its numerical analysis is also somewhat easier. Hence, we will concentrate on the applications of Eq. (14) in this article, leaving the implementation and applications of Eq. (15) for future studies.

3. Approach to the solutions

Although we have developed a formulation that should be solvable directly, the nonlinear equation actually has many numerical subtleties. In fact, the nonlinearity of Eq. (14) makes some simple algebraic operations such as taking square roots highly tricky, in order not to create artificial singularities or miscount zero-crossings in the process. Detailed analytic and numerical investigations of the equation are beyond the scope of this study.

In the present article, we will turn to an implementation of the above model in terms of optimization based on the algorithm called Null-space pursuit [16]. The method of Null-space pursuit is utilized only because it is a developed convenient tool and allows for a quick adaptation into a new Matlab program. The underlying principle here is still the model, namely Eqs. (13) and (14), that we have been discussing, and conceptually it is independent from the specific numerical realization. Also, in our implementation here the algorithm has been adapted according to the model so that our realization actually is quite different from the original scheme in [16]. Now, a general abstract form of the signal at hand is as follows:

$$S(t) = \varphi(t) + U(t) \tag{16}$$

where $s(t)$ is the observed signal, $\varphi(t)$ is the wanted (but unknown) component that represents the fastest oscillating part of the signal that permits a narrow-banded representation, and $U(t)$ is the residual. Basically, we are looking for a function that comes from the solution space of a second order differential equation:

$$\left(\frac{d^2}{dt^2} + \alpha(t)\right)\varphi(t) = 0, \quad \alpha(t) = \frac{\left[S'(t)\right]^2 + \left[\mathcal{H}\left[S'(t)\right]\right]^2}{\{F(t)\}^2 + [\mathcal{H}[F(t)]]^2} \cong \frac{\left[F'(t)\right]^2 + \left[\mathcal{H}\left[F'(t)\right]\right]^2}{\{F(t)\}^2 + [\mathcal{H}[F(t)]]^2}, \tag{17}$$

where $\alpha(t)$ is essentially the square of the instantaneous frequency. In the original Null Space Pursuit, $\alpha(t)$ will be first given by an initial guess (estimated mainly from zero crossings of the signal) and then act as a part of the trial parameters to be updated purely according to optimization and fitting purposes. Here, however, $\alpha(t)$ is a condition that will be forced upon the trial solution. Once a trial solution is obtained, a corresponding $\alpha(t)$ is then automatically decided, and substituted into the equation, while the initialization is done by using the original signal. In other words, in the current approach we do not optimize the equation itself but, instead, insist on the physical guiding principle and the definition of instantaneous frequency according to our model. Therefore, the setting is now substantially different from the perspective of optimization. The problem can then be expressed as:

$$\min_{U,\lambda_1,\lambda_2,\gamma} \left\{ \left\| \left(\frac{d^2}{dt^2} + \alpha(t)\right)(S-U) \right\|^2 + \lambda_1 \left(\|U\|^2 + \gamma \|S-U\|^2 \right) + \lambda_2 \|D\alpha(t)\|^2 \right\}. \tag{18}$$

Notice that in this formulation what is returned as an optimal solution is actually the residual part, and the wanted component is $(S-U)$. The inclusion of the parameters λ_1 and γ means we are somewhat relaxing the requirement that the component being strictly in the solution space. The parameter λ_2 is used to enforce smoothness on the instantaneous frequency, in order to better fulfill our initial assumption. The actual value of it, however, is not important, as numerical experiments have shown that the results are essentially insensitive to changes in λ_2. Therefore, it will be set to a nonzero constant. Also, the inclusion of λ_1 and γ, means that, unlike the original form of our model which will always solve for the fastest oscillating component first, the current procedure can be adapted to favor the component with greater power.

4. Simulation results

Some worked examples will be presented in the following to demonstrate that our model works excellently not only for simulated data but also for real physiological data. We will compare the results of the new method proposed with those of EMD (see more discussion in Section 5). In addition, we will also demonstrate that our new working definition for instantaneous frequency (based on Eq. (13)) is often both more stable and realistic.

4.1. Mixing of simple harmonic and oscillations with varying frequencies

In the first example, the input signal is a linear combination of three functions with two of them having time-dependent instantaneous frequencies and one with constant frequency (see Fig. 1). The two components with fluctuating frequencies relatively close are meant to simulate nonlinear signals, and the simple harmonic signal has a constant frequency that is reasonably separable from the spectra of the other two nonlinear components. Thus, it is natural to expect the decomposition should consist of three components corresponding to the input ones, respectively. However, since the designed frequency fluctuations of the first two components will incur some overlap between the spectra, it is in fact natural to expect some leakage between those two components. The resulting components obtained from the proposed method are close to the designed input signals, but not a perfect match, just as we expected, while the EMD results show somewhat more significant deviations from the input signals (see Figs. 2 and 3). This simulation does not necessarily mean that the new method can always outperform EMD because there could be situations where most of the components should be combined to preserve a special pattern. Nevertheless, if such a difference between the two methods is observed in real data, it perhaps should be interpreted as a sign calling for further investigation.

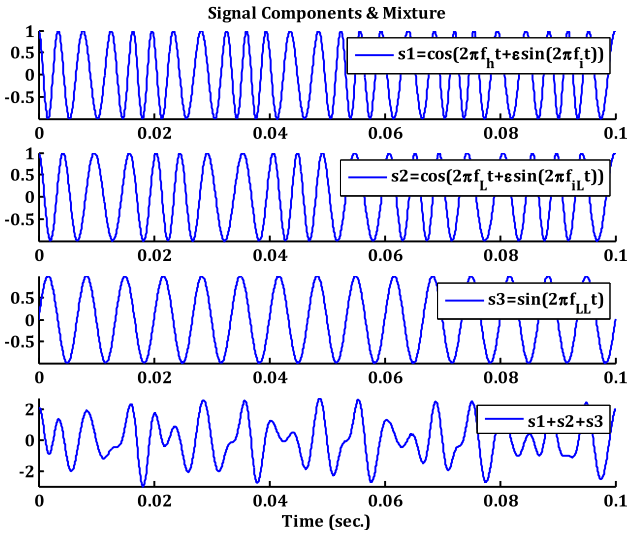

Fig. 1. The input signal (a combination of three simple harmonic functions) of our first numerical example. $f_h = 250, f_i = 55, f_L = 200, f_{iL} = 45, f_{LL} = 150, \varepsilon = 1$.

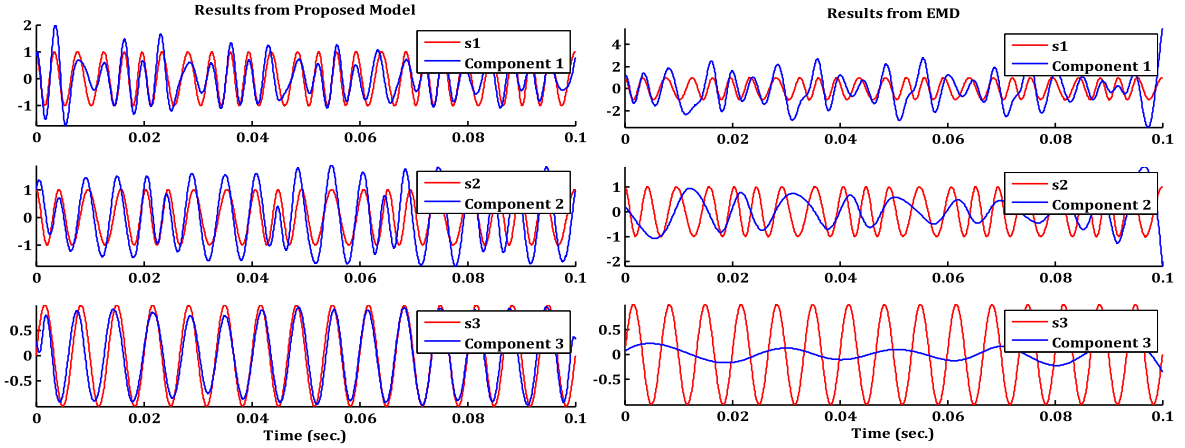

Fig. 2. The left column displays the results of our model, while the right those of EMD.

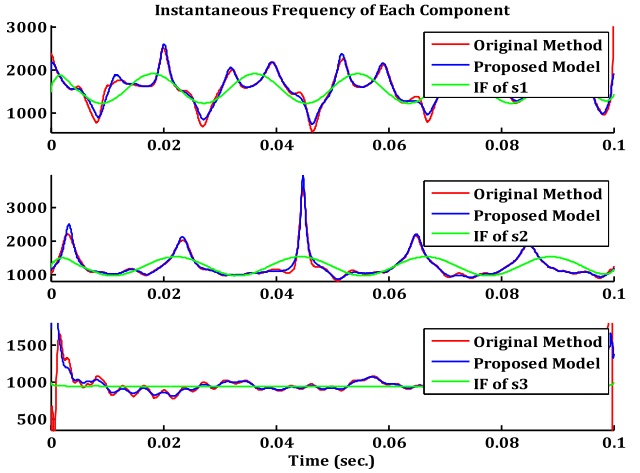

Fig. 3. The red and the blue represent instantaneous frequencies of the output components obtained by our method as in the left half of Fig. 2 estimated using two different methods (the original method means the traditional way of calculating instantaneous frequency by using Hilbert transform and inverse trigonometric functions, the proposed method here means estimation using Eq. (13)), respectively, while the green ones are the given (constant) frequency of the input components. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

4.2. Two simple harmonics with very close frequencies

The input signal used in the second example is a linear combination of two simple harmonic functions with very close frequencies (see Fig. 4). This kind of signal is often encountered in the studies of circadian research [17–19]. Here we have idealized the input signal, using functions with exact constant frequencies, whereas in the case of real data, the signals will exhibit more subtle behaviors, in particular the frequency would not be constant but having small fluctuations and, of course, there will be noises in the observed signals. Nevertheless, this example still demonstrates the essential features of and differences between the proposed method and EMD. As seen in Fig. 5, EMD attributes most of the mixed signal to the first IMF, and the second IMF looks like a much smaller residual signal. In contrast, the proposed model will produce two components with instantaneous frequencies mildly fluctuating around the input frequencies (Figs. 5 and 6), and with similar powers as the two designed components in the input signal. This example, however, perhaps should not be interpreted as that the new method had outplayed EMD. There might be scenarios where the components with near frequencies should be included in one component, to preserve some subtle and maybe substantial patterns. On the other hand, if it is necessary to distinguish and separate components oscillating at near frequencies, then the proposed model would provide a more accurate and efficient assessment.

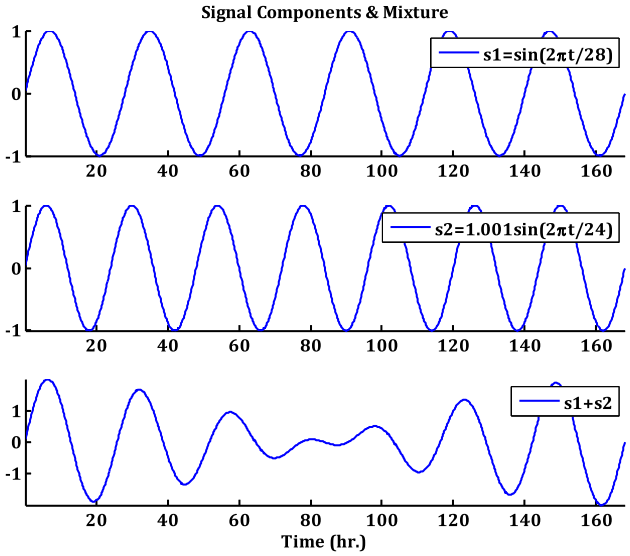

Fig. 4. The input signal and its two components used in the second example.

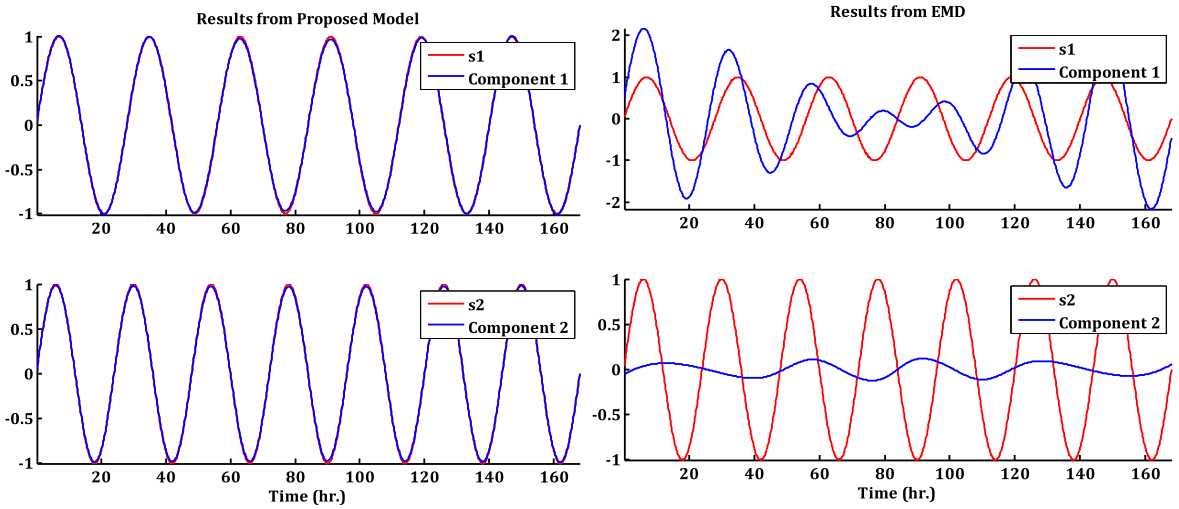

Fig. 5. Comparison between the input components and the extracted ones by the proposed model (left) and EMD (right).

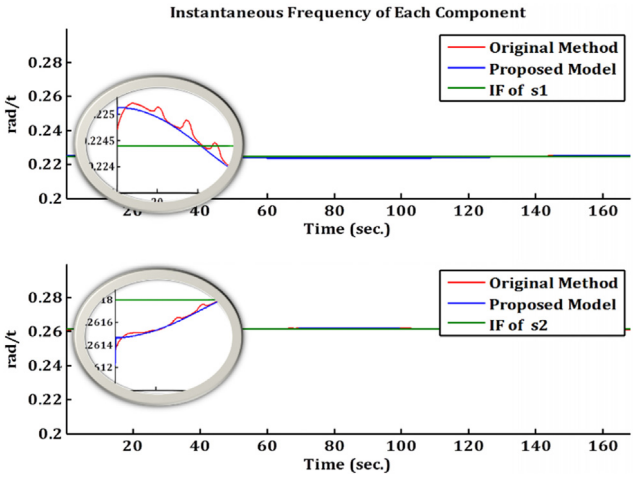

Fig. 6. The instantaneous frequencies of the components in Fig. 5 obtained by the proposed model, calculated using both the old and the new methods (see the caption of Fig. 3 for the definitions of the terms original method and proposed method).

4.3. Combination of CPR and VF signals

In the third example, the input signal is constructed using real data collected clinically. The aim we have in mind is to continually monitor heart rhythms under strong external disturbance, such as the operation of Cardiopulmonary Resuscitation (CPR). One such scenario is monitoring the ventricular fibrillation (VF) patterns to decide if a defibrillation is adequate [20]. Therefore, the input is a CPR signal plus a VF signal that was collected from actual patients and naturally contained noises. The VF signal is characteristic of turbulent and complex fractionated behaviors, with typically a broadband spectrum. Thus, a feasible strategy is to concentrate on accurately extracting the more regular CPR signal [11], and analyze the VF signal together with the residual. In this case, the proposed method has a largely satisfactory performance. While EMD can also achieve similar results if we combine a few appropriate IMFs to recover the CPR signal, the proposed model has produced slightly better results without having to examine and combine some output components afterwards (Figs. 7–9). Although in this case EMD seems to be somewhat less efficient, the fact that we need to analyze and combine a few IMFs after the decomposition actually hints on a finer analysis, had the disturbance been of more physiological importance.

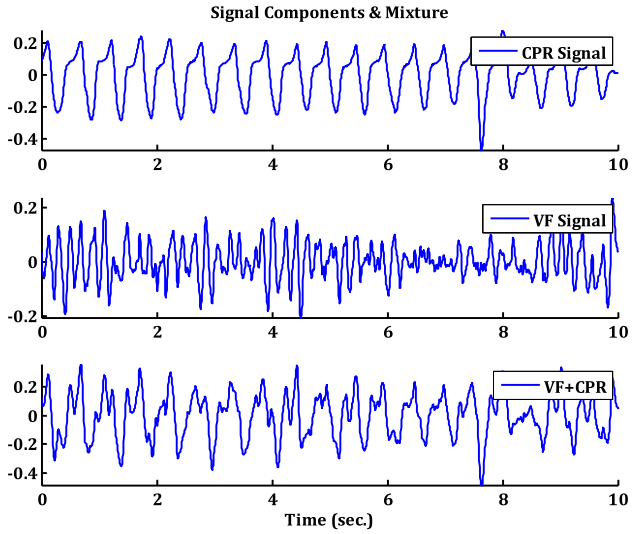

Fig. 7. The signal used in the third example, which is a mixture of real VF and CPR signals, collected from actual patients.

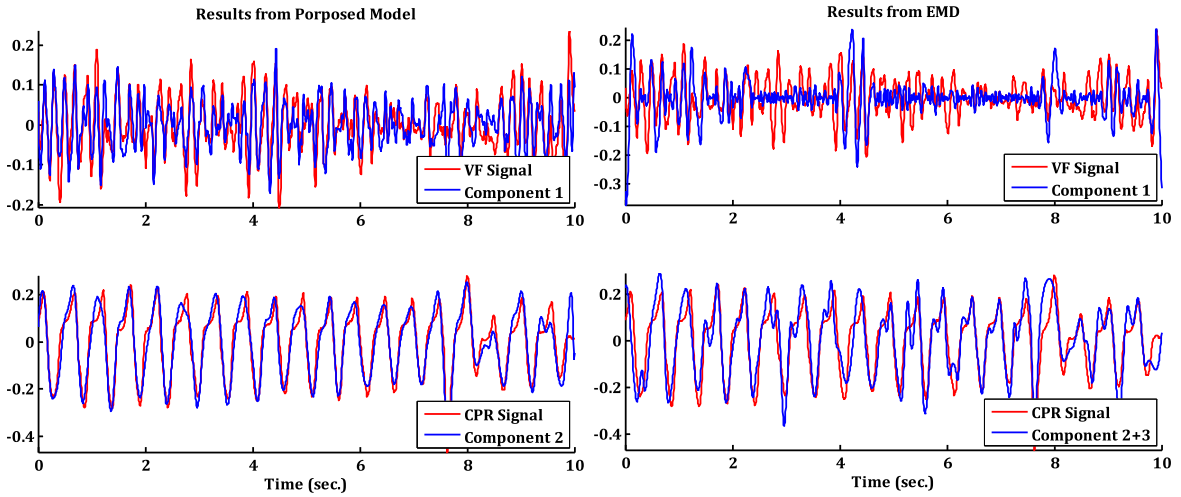

Fig. 8. Comparison between the input components and the extracted ones, obtained by using the proposed model (left) and EMD (right). Note that in the case of EMD the CPR signal is estimated using a combination of two IMFs.

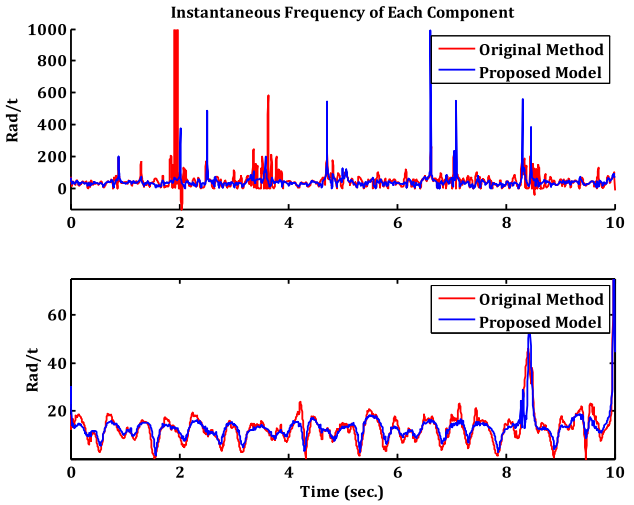

Fig. 9. The instantaneous frequencies of the components obtained by our method as in the left half of Fig. 8 (top: VF, bottom: CPR, and please see the caption of Fig. 3 for the definitions of the terms original method and proposed method for estimating the instantaneous frequency).

5. Conclusion and future work

First of all, one thing that is worth emphasis is that the sparse representations of signals that are proposed and studied in the context of nonlinear and adaptive signal analysis are analogous to, but essentially different from those in, say, compressed sensing [21–23]. What is important here is not a mathematical assumption that a given signal is sparse with a particular basis, nor is it a probabilistic statement that a sampling method will produce a sparse representation with small error. Rather, what we are seeking is using a few components (wave-packets) to characterize a complex time-varying data set. A given time series is often represented by a (complex-valued) distribution over the time–frequency plane after we apply time–frequency analysis methods [4,24–28], and to find a sparse representation is equivalent to finding a few characteristic curves on the two-dimensional time–frequency plane that can capture and reveal the information. In a sense, it is really a dimensional reduction that can make things more transparent. Our choice and development of the model in this paper are not derived from mathematical justification or algorithmic considerations. It is rather based on the belief of reasonable physical assumptions that natural signals from physical self-sustaining sources will fulfill. Specifically, oscillatory components, which are the signals that are narrow band and exhibit quasi-periodic behaviors with approximately zero local means, can be represented using AM–FM formalism; and the amplitude and the (instantaneous) frequency usually exhibit relatively slow and less violent changes compared with the phase. That said, we can also readily deduce that not all signals are suitable for such sparse decompositions, as there could be some kind of self-similarity or strong correlations throughout the whole observed frequency range, such that no set of curves can represent the data without losing some substantial

information. Again, it is worth emphasizing that most of the signals observed in medical and physiological research do satisfy our assumptions and can be viewed as the combination of several components, each from a self-sustaining source with distinct physical characteristics. However, even if the given signal has some kind of sparse representation, which tool is the most effective for the analysis still depends on the context and various practical limitations of the research problem. There can hardly be a single method that is ideal in every circumstance. The reasons for choosing EMD as the main target of our comparative analysis are mainly twofold: first, it is the most widely and successfully applied decomposition method, and it can be implemented to be computationally efficient and practical [29,30]. Second, EMD is the most well known representative of sparse decomposition methods and serves as the inspiration and benchmark to later developments [6–9,29,31]. The latter point deserves more explanation: while EMD has been very useful in many studies, the structure of itself from a theoretical perspective has not been investigated with much success. This is due to its highly nonlinear feature and also to the nature of EMD as being rather an algorithmic method than an analytically defined one. Although recently there has been some progress towards this end [31], much is still unexplored. Thus, part of the original motivation of the theory presented here is to shed some light on the physical meanings and possibly a mathematically clear formulation for sparse decomposition methods, in particular EMD. Having that in mind, the current proposed method has at least three valuable features. First, it is truly adaptive and can be applied to a wide class of signals without any stringent and artificial assumptions. In addition, although the model equations were derived under the assumption that the fluctuations of the instantaneous frequency and the envelope function are negligible, we have found that the application of the proposed model is not limited to this case. The reason is essentially that implementation of the model is really an optimal expansion using functions that are approximate solutions of a self-consistent nonlinear wave equation. In other words, this model, when put into practical use, is some kind of nonlinear eigenfunction expansion, and the original assumptions of widely separable frequencies and negligible envelope fluctuations are inessential, and thus would not restrict successful applications of the method. Last but not least, this model is more explicit and well-defined both mathematically and physically, which will make further finer studies and improvement much easier. Some of the ongoing and future work about this model will be addressed in the following.

The first and immediate extension of the current work would be an analytical study of the model equations. As mentioned, the nonlinearity and intricacy of the equations make a direct application of numerical solutions somewhat difficult and unstable. However, the equations themselves are interesting mathematically *per se*, and also, perhaps more importantly, some kind of skilled approximation or asymptotic analysis might lead to new physical meanings and simplified effective equations. In addition, we have modified and extended the model equations to the cases where the fluctuations of instantaneous frequency are considerably faster and larger, and currently the modified equations are being tested to see if they can provide more precision or physical insight. Also, in the current study we have utilized and modified the scheme of Null Space Pursuit to transform the decomposition problem into an optimization setting. While this development has been quite successful in many aspects, there are certainly other optimization schemes that might be more reasonable from a physics point of view, and can take advantage of the full power of our model. In fact, we have derived one such scheme and are currently transforming it to a more efficient computer program. The application and the performance of such schemes will be the subject of our future studies.

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